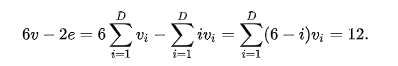
**Question:** A coin graph is a graph with vertices corresponding non-overlapping quarters (coins) and edges corresponding to touching pairs of coins

prove that any coin graph can be colored into 4 colors.

**Solution:**

1. In mathematics, the four color theorem, or the four color map theorem, states that, given any separation of a plane into contiguous regions, producing a figure called a map, no more than four colors are required to color the regions of the map so that no two adjacent regions have the same color.
2. Adjacent means that two regions share a common boundary curve segment, not merely a corner where three or more regions meet. It was the first major theorem to be proved using a computer.
3. If the four-color conjecture were false, there would be at least one-coin configuration with the smallest possible number of regions that requires five colors. The proof showed that such a minimal counterexample cannot exist, using two technical concepts:
   1. An unavoidable set is a set of configurations such that every coin configuration that satisfies some necessary conditions for being a minimal non-4-colorable triangulation (such as having minimum degree 5) must have at least one configuration from this set.
   2. A reducible configuration is an arrangement of coins that cannot occur in a minimal counterexample. If a coin placement contains a reducible configuration, then the configuration can be reduced to a smaller configuration. This smaller configuration has the condition that if it can be colored with four colors, then the original map can also. This implies that if the original map cannot be colored with four colors the smaller map can't either and so the original map is not minimal.
4. Suppose v, e, and f are the number of vertices, edges, and regions (faces). Since each region is triangular and each edge is shared by two regions, we have that 2e = 3f. This together with Euler's formula, v − e + f = 2, can be used to show that 6v − 2e = 12.
5. If *vn* is the number of vertices of degree *n* and *D* is the maximum degree of any vertex,



But since 12 > 0 and 6 − *i* ≤ 0 for all *i* ≥ 6, this demonstrates that there is at least one vertex of degree 5 or less.

1. If there is a graph requiring 5 colors, then there is a minimal such graph, where removing any vertex makes it four-colorable. Call this graph G. Then G cannot have a vertex of degree 3 or less, because if d(v) ≤ 3, we can remove v from G, four-color the smaller graph, then add back v and extend the four-coloring to it by choosing a color different from its neighbors.
2. Thus, any coin graph can be four-colored.

**Alternative proof:**

1. Consider the following set of propositional variables {Pn,i : 1 ≤ i ≤ 4 ∧ n∈N }. We are interpreting Pn,i as the nth coin has color i. Let Σ be the following set of sentences:
   1. Pn,1 ∨ Pn,2 ∨ Pn,3 ∨ Pn,4 ∀ n∈N, - says that every coin gets a color,
   2. ¬(Pn,i ∧ Pn,j), ∀ 1 ≤ i < j ≤ 4 and n∈N, - says that each coin gets at most one color
   3. ¬(Pn,i ∧ Pm,i), ∀ 1 ≤ i ≤ 4 and all pair of adjacent coins Cn and Cm - says that no two adjacent coins get the same cloud.
2. ΣΣ is finitely satisfiable by hypothesis, so by compactness, is satisfiable. Any truth valuation witnessing gives the decided coloring.